

## 6300 Signed Binary Representation of Integers

Computing  $a^x \bmod n$  for large integers  $x$  and  $n$  is the most time-consuming process in most public-key cryptography systems. An algorithm to compute  $a^x \bmod n$  can be described in a C-like pseudo-code as follows.

**Input:** positive integers  $a$ ,  $x$  and  $n$ ,

**Output:**  $y = a^x \bmod n$

**Method:**

```
convert  $x$  into binary  $(x_{k-1}x_{k-2}\dots x_0)_2$ ;  
 $y = 1$ ;  
for  $(i = k - 1; i \geq 0; i = i - 1)$  {  
     $y = y^2 \bmod n$ ;  
    if  $(x_i == 1)$   $y = y \times a \bmod n$ ;  
}  
print  $y$ ;
```

In the above algorithm, first  $x$  is converted into  $k$ -bit binary. Then the algorithm performs  $k$  iterations. In each iteration, a square ( $y^2 \bmod n$ ) is computed. In the  $i$ -th iteration, if  $x_i = 1$ , the algorithm also computes  $y = y \times a \bmod n$ . Therefore, the computing time depends on the number of 1's in the binary representation of  $x$ .

Let  $a^{-1}$  be the inverse of  $a$  in the group  $\mathbf{Z}_n^*$ . That is  $a \times a^{-1} \equiv 1 \pmod n$ . For example, assume that  $n = 13$ , then the inverse of 2 is 7,  $2 \times 7 = 14 \equiv 1 \pmod{13}$ . In this problem, you do not need to know how to compute the inverses.

Assume that  $a^{-1}$  is known, and  $x$  is represented by -1, 0, 1, instead of 0, 1, we can modify the above algorithm as follows.

**Input:** positive integers  $a$ ,  $x$  and  $n$ ,

**Output:**  $y = a^x \bmod n$

**Method:**

```
convert  $x$  into signed binary  $(x_{k-1}x_{k-2}\dots x_0)_{-1,0,1}$ ;  
 $y = 1$ ;  
for  $(i = k - 1; i \geq 0; i = i - 1)$  {  
     $y = y^2 \bmod n$ ;  
    if  $(x_i == 1)$   $y = y \times a \bmod n$ ;  
    if  $(x_i == -1)$   $y = y \times a^{-1} \bmod n$ ;  
}  
print  $y$ ;
```

In the above algorithm, we need to represent  $x$  by using -1, 0, 1. This is called *signed binary* representation of  $x$ . For convenience, we shall use  $\bar{1}$  to denote -1. For example:  $15 = (1111)_2 = (1000\bar{1})_{-1,0,1}$ .

You can see that it may be more efficient to use signed binary representation in computing  $a^x \bmod n$  when the inverse of  $a$  ( $a^{-1} \bmod n$ ) is known. For  $x = 15$ , using binary needs 4 multiplications, while using signed binary needs only 2.

In this problem, you are going to write a program to convert a large integer into signed binary. Signed binary representation of an integer may not be unique. We need a representation with minimum number of non-zero bits to make the computation of  $a^x \bmod n$  fast.

A hint of converting  $x$  into a signed binary with minimum number of non-zero bits: Make sure that no adjacent two bits are both non-zero.

### Input

The input to this problem is a sequence of large integers. Each integer  $x$  is no more than 120 decimal digits, and it is written in a line. There will be no spaces in front of  $x$ , and no spaces after  $x$ . The last line of the input file contains a single '0'. You do not need to process this line.

### Output

The outputs for each test case consists of two parts written in one line. The first part is an integer  $b$ , which is the number of non-zero bits of the binary representation of  $x$ . The second part is an integer  $s$ , which is the number of non-zero bits in the signed binary representation of  $x$ . Print exactly one space between these two parts.

### Sample Input

```
15
2514593
113113561845
0
```

### Sample Output

```
4 2
11 9
23 14
```