3352 The Rank of a Tree

Let B_n denote the set of binary trees with n internal nodes (and thus with $n+1$ leaves) and $T \in B_n$. We assume that all internal nodes of T are numbered from 1 to n, according to the in-order traversal of T. For each internal node $i \in T$, the left sub-tree of i is the sub-tree of T rooted at the left child of i. Then the *left-weight* of i, denoted by $w_l(T,i)$, is the number of leaves in the left sub-tree of i. The integer sequence $w_l(T) = (w_l(T,1), w_l(T,2), \ldots, w_l(T,n))$ is defined as the *left-weight sequence* of T and every binary tree can be characterized by its left-weight sequence. For example, Figure 6 shows a tree $T \in B_6$ with $w_l(T) = (1,2,1,4,1,2)$.

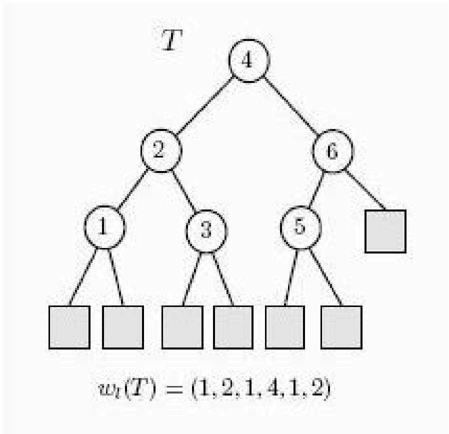

Figure 6: The left-weight sequence of a tree T.

The number of binary trees in B_n is given by the Catalan number

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

For example, all the left-weight sequences of $T \in B_4$ are listed in Table 1 in lexicographic order. The *rank* of a left-weight sequence is its ordinal number in the lexicographic order list of all left-weight sequences in B_n. For example, the rank of the tree $T \in B_4$ with $w_l(T) = (1,1,2,4)$ is 7.

Design a program for computing the rank of a specific binary tree $T \in B_n$.

Input

The input file consists of several test cases. Each test case starts with a positive integer n, $3 \le n \le 10$. Then followed by a line which contains the left-weight sequence of a specific binary tree. The line containing a '0' indicates the end of test cases.

Output

For each test case, output the rank of the given left-weight sequence in one line.

Rank	weight sequence
1	1111
2	1112
3	1113
4	1114
5	1121
6	1123
7	1124
8	1131
9	1134
10	1211
11	1212
12	1214
13	1231
14	1234

Table 1: Left-weight sequences of $T \in B_4$

Sample Input

```
5
1 1 1 1 2
8
1 2 3 1 1 2 1 8
10
1 1 2 1 5 1 2 1 4 1
0
```

Sample Output

```
2
1323
9000
```