

## 3220 Heapsort

A well known algorithm called *heapsort* is a deterministic sorting algorithm taking  $O(n \log n)$  time and  $O(1)$  additional memory. Let us describe ascending sorting of an array of different integer numbers.

The algorithm consists of two phases. In the first phase, called *heapification*, the array of integers to be sorted is converted to a *heap*. An array  $a[1 \dots n]$  of integers is called a heap if for all  $1 \leq i \leq n$  the following *heap conditions* are satisfied:

- if  $2i \leq n$  then  $a[i] > a[2i]$ ;
- if  $2i + 1 \leq n$  then  $a[i] > a[2i + 1]$ .

We can interpret an array as a binary tree, considering children of element  $a[i]$  to be  $a[2i]$  and  $a[2i + 1]$ . In this case the parent of  $a[i]$  is  $a[i \text{ div } 2]$ , where  $i \text{ div } 2 = \lfloor i/2 \rfloor$ . In terms of trees the property of being a heap means that for each node its value is greater than the values of its children.

In the second phase the heap is turned into a sorted array. Because of the heap condition the greatest element in the heapified array is  $a[1]$ . Let us exchange it with  $a[n]$ , now the greatest element of the array is at its correct position in the sorted array. This is called *extract-max*.

Now let us consider the part of the array  $a[1 \dots n - 1]$ . It may be not a heap because the heap condition may fail for  $i = 1$ . If it is so (that is, either  $a[2]$  or  $a[3]$ , or both are greater than  $a[1]$ ) let us exchange the greatest child of  $a[1]$  with it, restoring the heap condition for  $i = 1$ . Now it is possible that the heap condition fails for the position that now contains the former value of  $a[1]$ . Apply the same procedure to it, exchanging it with its greatest child. Proceeding so we convert the whole array  $a[1 \dots n - 1]$  to a heap. This procedure is called *sifting down*. After converting the part  $a[1 \dots n - 1]$  to a heap by sifting, we apply extract-max again, putting second greatest element of the array to  $a[n - 1]$ , and so on.

For example, let us see how the heap  $a = (5, 4, 2, 1, 3)$  is converted to a sorted array. Let us make the first extract-max. After that the array turns to  $(3, 4, 2, 1, \mathbf{5})$ . Heap condition fails for  $a[1] = 3$  because its child  $a[2] = 4$  is greater than it. Let us sift it down, exchanging  $a[1]$  and  $a[2]$ . Now the array is  $(4, 3, 2, 1, \mathbf{5})$ . The heap condition is satisfied for all elements, so sifting is over. Let us make extract-max again. Now the array turns to  $(1, 3, 2, \mathbf{4}, \mathbf{5})$ . Again the heap condition fails for  $a[1]$ ; exchanging it with its greatest child we get the array  $(3, 1, 2, \mathbf{4}, \mathbf{5})$  which is the correct heap. So we make extract-max and get  $(2, 1, \mathbf{3}, \mathbf{4}, \mathbf{5})$ . This time the heap condition is satisfied for all elements, so we make extract-max, getting  $(1, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5})$ . The leading part of the array is a heap, and the last extract-max finally gives  $(\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5})$ .

It is known that heapification can be done in  $O(n)$  time. Therefore, the most time consuming operation in heapsort algorithm is sifting, which takes  $O(n \log n)$  time.

In this problem you have to find a heapified array containing different numbers from 1 to  $n$ , such that when converting it to a sorted array, the total number of exchanges in all sifting operations is maximal possible. In the example above the number of exchanges is  $1 + 1 + 0 + 0 + 0 = 2$ , which is not the maximum.  $(5, 4, 3, 2, 1)$  gives the maximal number of 4 exchanges for  $n = 5$ .

### Input

Input file contains several datasets. Each consists on a single line with the number  $n$  ( $1 \leq n \leq 50\,000$ ).

**Output**

For each test case, output the array containing  $n$  different integer numbers from 1 to  $n$ , such that it is a heap, and when converting it to a sorted array, the total number of exchanges in sifting operations is maximal possible. Separate numbers by spaces.

**Sample Input**

6

**Sample Output**

6 5 3 2 4 1