2271 Hypercube Routing

A key component of Parallel Computer architecture is the Interconnection Network. This network enables the communication among different processors of the computer. Throughout the years, different interconnection networks have been used. One of the most popular and efficient networks is the hypercube network. This network has been used in different systems such as nCube, Intel IPSC, Thinking Machines CM-2 and the SGI-Gray Origin 2000 supercomputer.

A hypercube network of dimension \( k \) has the following properties

- The total number of nodes is \( 2^k \).
- The nodes are identified with a node number in the range \( 0..2^k - 1 \)
- Every node has \( k \) bi-directional links with other nodes.
- Two nodes are connected if and only if the binary representation of the node number of the two nodes differs in exactly 1 bit.
- The binary representation of nodes in a hypercube of dimension \( k \) has \( k \) bits (the exception is the singular case of hypercube with \( k = 0 \)).

The following figure shows a graphical representation of hypercube of dimensions 0, 1, 2 and 3.
A shortest path between any two nodes of a hypercube is represented as a sequence of nodes $N_0, N_1, \ldots, N_J$, where $J$ is the length of the path. In a hypercube, there might be more than one shortest paths between any two nodes. We are interested in computing a bit-position ordered shortest path. A bit-position ordered shortest path is a hypercube shortest path with the following property:

For any pair of nodes $N_i, N_{i+1}$ that belong to the sequence that represents a shortest path, if the difference in the bit representation of nodes $N_i$ and $N_{i+1}$ is in bit position $q$ (bit position is considered from right to left) then, the bit position of the difference in the bit representation between any pair of nodes that are in the sequence before $N_i$ should be less than $q$. Also the bit position of the difference in the bit representation between any pair of nodes that are after $N_{i+1}$ in the sequence should be greater than $q$.

For example, for hypercube with $k = 3$, a bit-position ordered shortest path between nodes 1(001) and 6(110) is 1(001), 0(000), 2(010) and 6(110).

A shortest path that is not bit-position ordered is 1(001), 3(011), 7(111) and 6(110). The problem here is that nodes 1 and 3 differ in bit position 2 however, nodes 7 and 6 differ in bit position 1.

Write a program that computes the bit-position ordered shortest path between any two nodes of the hypercube and also outputs the length of the path.

**Input**

Your program will repeatedly accept the input sets from the input file.

Each input set has the dimensionality of a hypercube (the dimension of the hypercube is in the range of 0 up to 20) a source and a destination node (integer numbers in decimal base). The program will stop when it identifies an input data set with dimension 0.

**Output**

The program will output the following information:

- The length (number of links that must be traversed) of the shortest path between the source and the destination nodes.
- An ordered shortest path showing all the nodes that are visited from the source to the destination. The nodes must be shown in binary and decimal representation.

Print a blank line between sets.

**Sample Input**

2 0 3  
5 20 6  
3 2 5  
0

**Sample Output**

Hypercube Dimension 2  
Source Node 0  
Destination Node 3  
Shortest Path Length 2  
Shortest Path  
00 (0)
Hypercube Dimension 5
Source Node 20
Destination Node 6
Shortest Path Length 2
Shortest Path
10100 (20)
10110 (22)
00110 (6)

Hypercube Dimension 3
Source Node 2
Destination Node 5
Shortest Path Length 3
Shortest Path
010 (2)
011 (3)
001 (1)
101 (5)